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NATIONAL DEFENSE RESEARCH COMMITTEE

ARMOR AND ORDNANCE REPORT NO. A-227 (OSRD NO. 2043)

DIVISION 2

THE MECHANICS OF ARMOR PERFORATION

I. Residual Velocity

by

H. P. Robertson

A Reissue of NDRC Report No. A-16 (OSRD No. 19)
with Corrections and with an Addendum
by A. H. Taub and C. W. Curtis

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A Reissue of NDRC Report No. A-16 (OSRD No. 19)
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by A. H. Taub and C. W. Curtis

Approved on November 10, 1943
for submission to the Division Chief

Walker Bleakney

Walker Bleakney, Head
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for submission to the Committee

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[redacted]

Preface

The work described in this report is pertinent to the projects designated by the War Department Liaison Officer as CE-5 and CE-6, to the project designated by the Navy Department Liaison Officer as NO-11 and to Division 2 project P2-101.

This work was carried out at Princeton University as part of its performance under contract OEMsr-260.

This report was originally issued in July 1941 as NDRC Report A-16 (OSRD No. 19) and was based upon a study on penetration mechanics from results made available by the Naval Proving Ground and by the Naval Research Laboratory. It was to be considered as Part I of a theoretical study of the mechanics of armor perforation. Further parts, then under preparation, were Part II, "Limit velocity," and Part III, "Resisting force during the penetration cycle."

"The study was initially undertaken because of its possible relationship to the problems of interest to the Committee on Passive Protection Against Bombing -- now the Committee on Fortification Design -- of the National Research Council; much of the material used was obtained in contacts made possible by that committee. Thanks are also due to the officers and civilians of the Army and Navy who assisted with their resources and advice and to R. J. Slutz for assistance in computation and in preparing figures.

The issuance of Parts II and III was delayed by the necessity, when the United States entered the war, of working on problems of greater urgency or more immediate applicability. Part III was issued as NDRC Report A-211 (OSRD No. 1798), The mechanics of armor perforation, III, resisting force during the penetration cycle, by H. P. Robertson. However, as far as Part II is concerned, such partial results as had been attained were for the most part incorporated into NDRC Report A-111 (OSRD No. 1027), The ballistic properties of mild steel, by the Ballistic Research Group, Princeton University. NDRC Report A-156 (OSRD No. 1301), Ballistic tests of STS armor plate, using 37-mm projectiles, by the Ballistic Research Group, Princeton University, also contains newer experimental data.

This reissue of Report A-16 contains an addendum by A. H. Taub and C. W. Curtis and also a number of minor corrections.

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The NDRC technical reports section
for armor and ordnance edited
this report and prepared it for duplication.

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THE MECHANISM OF ARMOR PENETRATION

I. Residual Velocity

Abstract

1. It is shown that, on the Poncelet-Morin theory of the resistance offered a projectile in motion through a dense medium, the residual energy E_r of the projectile on emerging from a plate is a linear function,

$$E_r = s(E_s - E_l), \qquad \text{(i)}$$

of its striking energy E_s, and (in Appendix A) that this linear dependence is characteristic of any resistance,

$$R = \alpha + \beta E, \qquad \text{(ii)}$$

which is linear in the instantaneous energy $\underline{E}$ of the projectile. It is also shown (in Appendix A) that, subject to certain limitations, the inverse holds; namely, that if the residual energy E_r is found experimentally to be a linear function of the striking energy E_s, as in Eq. (i), then the resistance $\underline{R}$ at any stage in the penetration cycle must depend linearly on the instantaneous energy $\underline{E}$ of the projectile, as in Eq. (ii). It is believed that the study of this dependence offers a tool which should prove of theoretical value in testing proposed mechanisms of perforation, and of practical value in enabling the determination of plate limit from even a single shot at a velocity above the limit velocity -- as suggested in the letter1/ referred to in Sec. 2(a) of this report.

2. Data on major caliber projectiles, obtained from the Naval Proving Ground, are analyzed in accordance with the procedure proposed in Sec. 1. It is found possible to represent these data in terms of a linear relationship (i) between residual and striking energy and to deduce therefrom plate limits v_l that are in good agreement with those obtained at the Naval Proving Ground from these and other data.

3. Data on small caliber projectiles, furnished by the Naval Research Laboratory, are subjected to the same analysis. These data, which are by-products of a study of resistance-penetration relations, are found to straggle more than the major caliber data but are not inconsistent with the present hypothesis. (Small caliber tests, in which efforts are directed solely toward obtaining data of relevance here,

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1/ Naval Proving Ground letter S13-1(7), Mar. 31, 1928.

are in progress under the auspices of Section S, Division A, NDRC and will be reported in due course by those responsible;[2] a preliminary analysis of the data so far available, along the present lines, indicates good agreement with the general predictions of the theory here proposed.)

4. A summary of recent developments in the theory of the mechanism of armor penetration will be found in Appendix B at the end of the report. The evidence now substantiates the qualitative ideas expressed in the original report, but quantitatively there are still some discrepancies.

1. Basic theory

An attempt was made in "Terminal Ballistics"[3] to describe the penetration cycle in terms of two elements that must contribute to the resistance encountered by a projectile moving through a dense medium; namely, (a) the resistance due to the cohesive forces of the target material, and (b) the inertial resistance of the resulting detritus. The equation of motion was accordingly taken to be

$$m\,dv/dt = -A(x,e)(a + \tfrac{1}{2}\gamma\rho' v^2), \qquad (1.1)$$

where m and v are the mass and velocity of the projectile; $A(x,e)$ is the "area of impression" of the projectile when its tip has penetrated a distance x into the face of a plate of thickness e; ρ' is the density, or mass per unit volume, of the target material; and, finally, a is the "shatter coefficient" and γ the "inertial coefficient," which complete the specification of the two elements here

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2/ Since this manuscript was written the following reports have been issued: NDRC Report A-67 (OSRD No. 689) Ballistic tests of small armor plates for the Frankford Arsenal, by G. T. Reynolds, R. L. Kramer and W. Bleakney; NDRC Report A-111 (OSRD No. 1027), The ballistic properties of mild steel, including preliminary tests on armor steel and dural, by the Ballistic Research Group, Princeton University; NDRC Report A-156 (OSRD No. 1301), Ballistic tests of STS armor plate, using 37-mm projectiles, by the Ballistic Research Group, Princeton University.

3/ H. P. Robertson, Interim report of Committee on Passive Protection Against Bombing to the Chief of Engineers, U.S. Army; hereinafter referred to as TB-I.

taken into account. The velocity $\underline{v}$ at any depth $\underline{x}$ is then found by integration of Eq. (1.1) to be

$$v^2 = \varepsilon^{-\gamma m'(x,e)/m}\,[v_s^2 - u^2(\varepsilon^{\gamma m'(x,e)/m} - 1)], \qquad (1.2)$$

where

$$m'(x,e) = \rho' \int_0^x A(x,e)\,dx, \qquad (1.3)$$

Here $m'(x,e)$ is the mass of target material displaced by the projectile at penetration distance $\underline{x}$ (neglecting warping of the plate), $\underline{\varepsilon}$ is the base of natural logarithms, v_s is the striking velocity of the projectile, and $u^2[= 2a/\gamma\rho']$ is a parameter having the dimensions of the square of a velocity.

The plate will be said to be <u>perforated</u> (completely) when the base of the projectile has left the rear face of the plate; its <u>residual velocity</u> v_r will then be given by Eq. (1.2) when $\underline{x}$ is large enough for the bullet to perforate the plate. The expression for v_r^2 then becomes

$$v_r^2 = \varepsilon^{-\gamma m'/m}(v_s^2 - v_l^2), \qquad (1.4)$$

where $\underline{m'}$ is $\pi(\frac{1}{2}d)^2 e\rho'$, the mass of a cylinder of diameter $\underline{d}$ (caliber) and height $\underline{e}$ cut out of the plate. The <u>limit velocity</u> v_l -- that is, the striking velocity required to cause perforation with residual velocity $v_r = 0$ -- is then given by

$$v_l^2 = u^2(\varepsilon^{\gamma m'/m} - 1). \qquad (1.5)$$

Although these results have been derived on the basis of the admittedly over-simplified assumptions embodied in Eq. (1.1) for the resistance $\underline{R}$, further theoretical considerations show that certain features of them hold under much broader assumptions; this is particularly true of the linear relationship between v_r^2 and v_s^2 expressed by Eq. (1.4). But, since it is the main purpose of the present report to analyze the empirical data available from Navy sources, the continuity of the development will not be interrupted by introducing at this point the frankly phenomenological and possibly ephemeral theory upon which this extension is based; this latter has therefore been relegated to Appendix A, where it may be consulted by those who are interested.

The data on residual velocity will therefore be examined from the standpoint of Eq. (1.4), that is,

$$v_r^2 = \varepsilon^{-z}(v_s^2 - v_l^2), \qquad (1.6)$$

where $\underline{z}$ is $\gamma m'/m$. Following the Naval Proving Ground procedure, we will express the limit velocity v_l in the more general form,

$$v_l = (e^{\frac{1}{2}}d/W^{\frac{1}{2}})F(z), \qquad (1.7)$$

where $\underline{W}[= mg]$ is the weight of the projectile in pounds; $\underline{F}$ is essentially the Thompson $\underline{F}$-coefficient, except that it is here considered as a function of $\underline{z}[=\gamma m'/m]$ instead of e/d, to which $\underline{z}$ is proportional.[4/] It will at times be found more convenient to express these formulas in terms of the kinetic energy $\underline{E}[= \frac{1}{2}mv^2]$ of the projectile, in place of its velocity $\underline{v}$. They then become

$$E_r = \varepsilon^{-z}(E_s - E_l), \qquad (1.8)$$

$$E_l = Ae\,P(z), \qquad (1.9)$$

where $\underline{A}[= \pi(\frac{1}{2}d)^2]$ is the cross-sectional area of the projectile, and

$$P(z) = (2/\pi g)\,F^2(z) \qquad (1.10)$$

is the "average pressure" of the plate-projectile reaction, as this term is used by the Naval Research Laboratory.

In the strict Poncelet theory given in TB-I,

$$F(z) = \left(\tfrac{1}{2}\pi a g\,\frac{\varepsilon^z - 1}{z}\right)^{\frac{1}{2}} = \sqrt{\tfrac{1}{2}\pi a g}\,\left(1 + \tfrac{1}{4}z + \frac{5}{96}z^2 + \ldots\right), \qquad (1.11)$$

$$P(z) = a\,\frac{\varepsilon^z - 1}{z} = a\left(1 + \tfrac{1}{2}z + \frac{1}{6}z^2 + \ldots\right), \qquad (1.12)$$

4/ See TB-I, Eq. (6.10).

where a is considered as independent of z; with this value of F(z), Eq. (1.7) agrees with Eq. (1.5). An eventual dependence of the shatter strength a on z (or on the relative thickness e/d of the plate) may be allowed, as suggested by the theoretical extension treated in Appendix A.

These formulas are all based on the assumptions of rectilinear path, normal impact and absence of yaw. In the case of major caliber projectiles no attempt is made to allow for "cap effect," so the F-values obtained are directly comparable with those given in Naval Proving Ground reports. The small caliber data are obtained from projectiles on which the jacket, and therefore any "jacket effect," has been greatly reduced.

Units. -- The units adopted throughout the present report are, in general, the same as those used in TB-I, namely,

x, e, feet	d, inches
t, seconds	v, feet per second
W, pounds	m = W/g, slugs
E, foot pounds	P, pounds per square inch.

The principal exception is encountered in computing the F-value corresponding to a limit velocity v_l, where, following Naval Proving Ground procedure, the caliber d is also to be measured in feet instead of inches; the relation between the numerical value of F thus computed and P (lb/in^2) is then

$$P(z) = (1/72\pi g)\,F^2 = 1.374 \times 10^{-4} F^2 \qquad (1.12)$$

Throughout the treatment the base of natural logarithms, 2.718..., is denoted by ε instead of by the conventional "e," to avoid confusion with the thickness e of the plate.

2. Residual velocity; major caliber

The principal point to be examined is that of the relationship between residual and striking velocities or energies. The data available at the present time are meager; however, the analysis of these data is favorable to the view that this relationship is, within the observational error, linear in the energies. In each set of data the limit velocity or energy and the slope ε^{-z} of the E_r,E_s-line are obtained, in accordance with Eq. (1.6) or (1.8), and the values of F and P are computed from Eqs. (1.7) and (1.9).

Expressed in graphical terms, the precise procedure adopted in reducing the data is as follows. The points S_i representing the individual shots have been plotted on an E_r,E_s-(or v_r^2,v_s^2-) diagram, as in Fig. 1. The best straight-line fit L to these points has been obtained

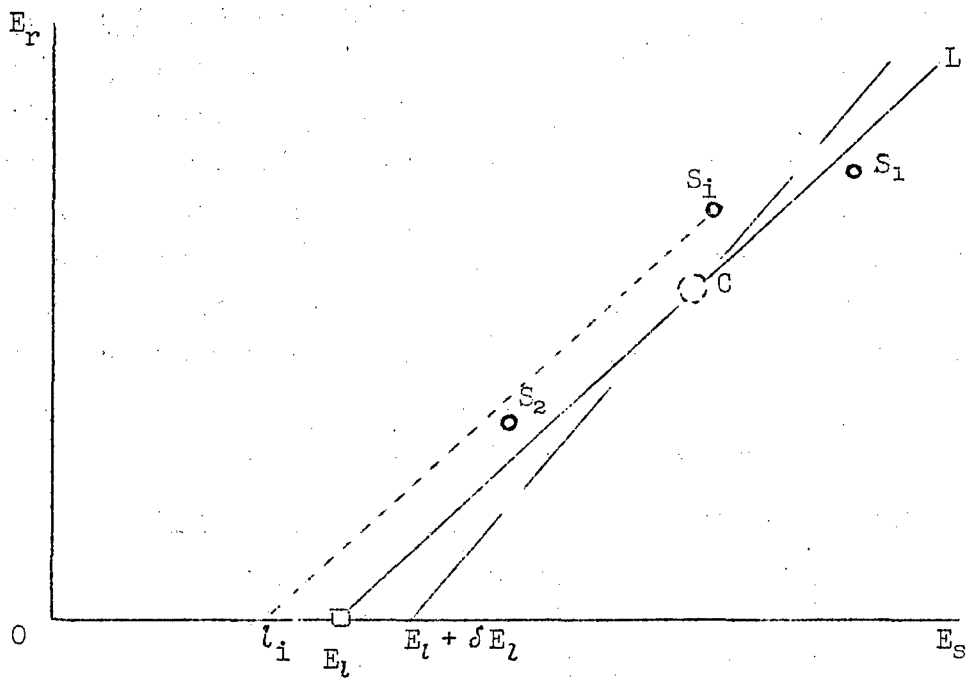

Fig. 1. E_r, E_s diagram.

by the method of least squares; specifically, by the requirement that Σd_i^2, the sum of the squares of the perpendicular distances d_i of the points S_i from L, be minimum. This determines the slope $s = \varepsilon^{-z}$ and the E_s-intercept E_l of the line L, and from them γ and F may be computed. It is to be noted that L must pass through the centroid C of the points S_i.

A convenient measure of the goodness of fit may be obtained by considering the distribution of the E_s-intercepts l_i of the lines drawn through the data points S_i parallel to $\underline{L}$; the "probable error" δE_l of the limit energy is then conventionally taken to be the probable error of these abscissas l_i. Some idea of the confidence to be placed in the slope $\underline{s}$, and hence in the inertial coefficient $\underline{\gamma}$, can then be had by considering the change δs in the slope on going over from $\underline{L}$ to a line passing through the centroid $\underline{C}$ and the point $E_l + \delta E_l$ on the E_s-axis.

It will be found, at least in the more consistent runs, that the slope $\underline{s}[= \varepsilon^{-z}]$ of the E_r, E_s-graph is somewhat less than 1. It follows from this that $\underline{z}$, and therefore $\underline{\gamma}$, is greater than zero, whence the resistance $\underline{R}$ encountered by the projectile increases with its velocity or energy. The limiting case, $s = 1$, corresponding to $\gamma = 0$, would arise if $\underline{R}$ were independent of velocity, for in this case the energy $E_s - E_r$ absorbed in the process would also be independent of the velocity -- or better, of the striking energy E_s. The dependence of the resistance on velocity is therefore determined essentially by the small deviation of the slope $\underline{s}$ from the critical slope 1; unfortunately this deviation is extremely sensitive to accidental irregularities in the data, so that it is only possible to conclude from the data here analyzed that $\underline{\gamma}$ is of the order 0.0 to 0.4.

(a) 12-in. projectile on 8-in. plate at normal incidence. --

References: NPG Letter S13-1(7), Mar. 31, 1928;
NPG Photo No. 3051, Mar. 1, 1928.

Projectile: Navy Standard AP 12-in., W = 870 lb.

Plate: 8-in. class B,
ρ'(assumed) = 0.283 lb/in.^{3}, W = 256 lb.

v_s	v_r	$v_s^2 \times 10^{-3}$	$v_r^2 \times 10^{-3}$	v_r(computed)
1337	574	1788	329	570
1516	884	2298	781	886
1695	1126	2873	1268	1144
1784	1286	3183	1654	1261
1895	1390	3591	1932	1400

The best straight-line fit to these data is that given by the equation,

$$v_r^2 = 0.907\,(v_s^2 - 1431 \times 10^3),$$

whence

$$v_l^2 = (1431 \pm 0.012) \times 10^3 \text{ ft}^2/\text{sec}^2, \quad s = 0.907 \pm 0.008.$$

From these values it follows that

$$v_l = (1196 \pm 5) \text{ ft/sec}, \qquad F = 43200 \pm 200,$$

$$P = (256000 \pm 2000) \text{ lb/in}^2 = (180 \pm 2) \text{ kg/mm}^2,$$

$$\gamma = 0.33 \pm 0.03.$$

In the aforementioned NPG reference it was concluded, from the data here used augmented by other (incomplete) perforations, that v_l was 1205 ft/sec.

In Fig. 2 the data and best fit are plotted in terms of v_r^2, v_s^2. In Fig. 3 they are plotted in accordance with NPG Photo No. 3051; that is $\Delta v[= v_s - v_r]$ and $\Delta v^2[= v_s^2 - v_r^2]$ are plotted as functions of v_s.

(b) 12-in. projectile on 3-in. plate at normal incidence. --

Reference: NPG Memo. S13-1(7) (B), Nov. 3, 1936.

Projectile: 12-in., W = 870 lb.

Plate: 3-in. class B (?),
ρ' (assumed) = 0.283 lb/in^3, W' = 96 lb.

v_s	v_r	$v_s^2 \times 10^{-3}$	$v_r^2 \times 10^{-3}$
955	718	912	515.5
1391	1234	1934	1523

The equation of the straight line passing through these two points is

$$v_r^2 = 0.986\,(v_s^2 - 389 \times 10^3),$$

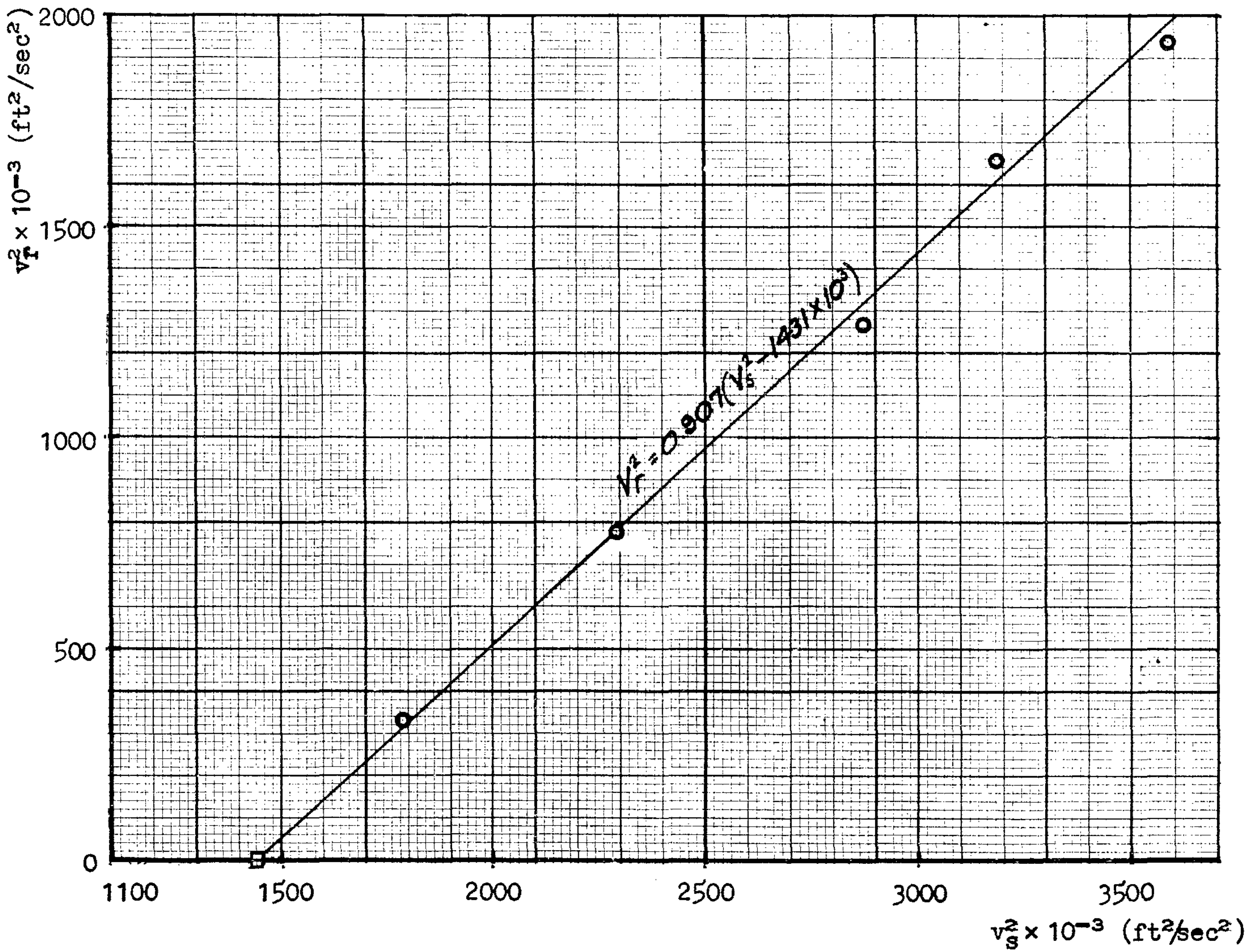

Fig. 2. Graph of data for 12-in. projectiles on 8-in. plate at normal incidence, plotted in terms of v_r^2 and v_s^2.

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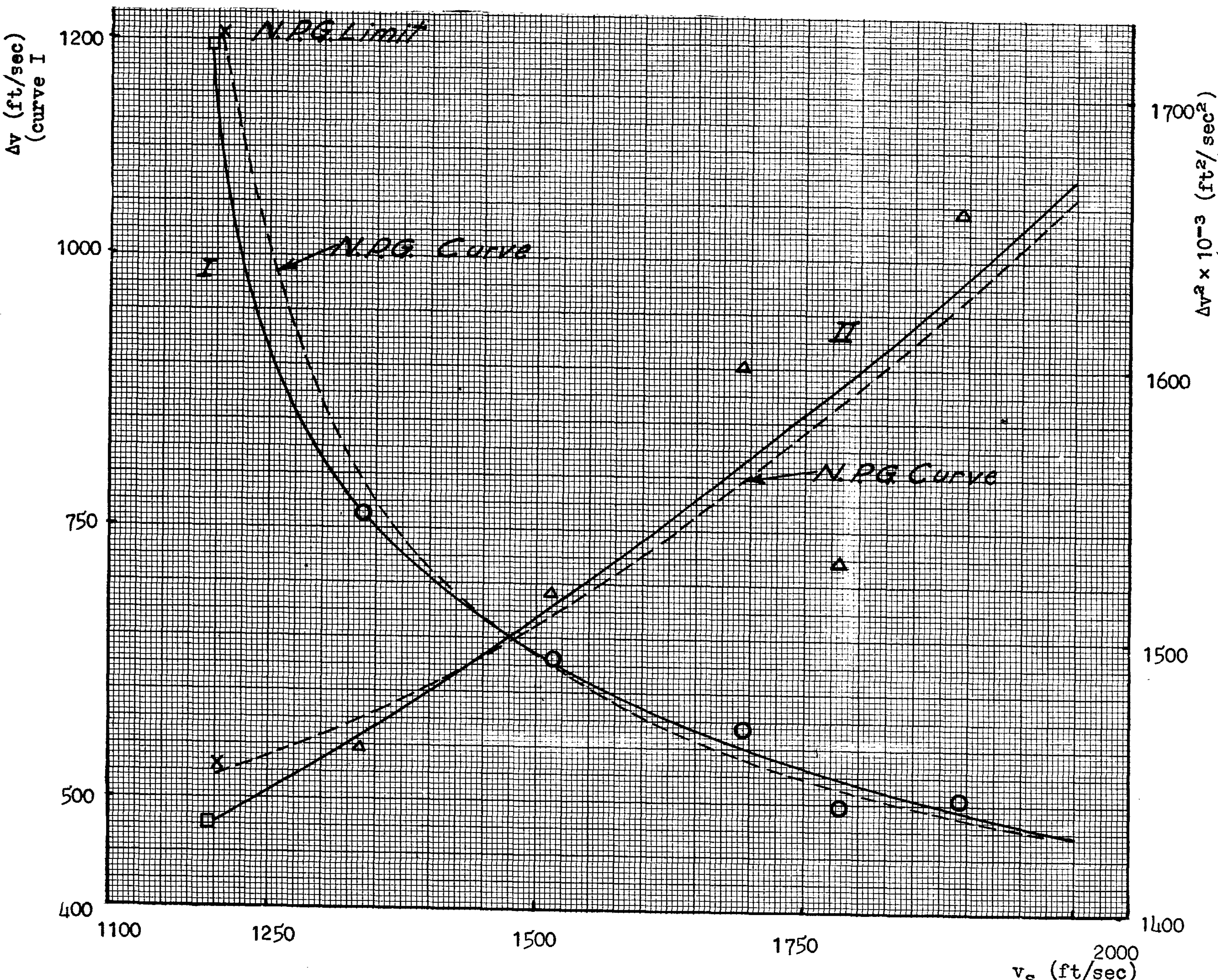

Fig. 3. Data for 12-in. projectiles on 8-in. plate at normal incidence. The dotted curves are plotted in manner of N.P.G. Photo No. 3051; curves I and II (full line) computed from equation $v_r^2 = 0.907\ (v_s^2 - 1431 \times 10^3)$.

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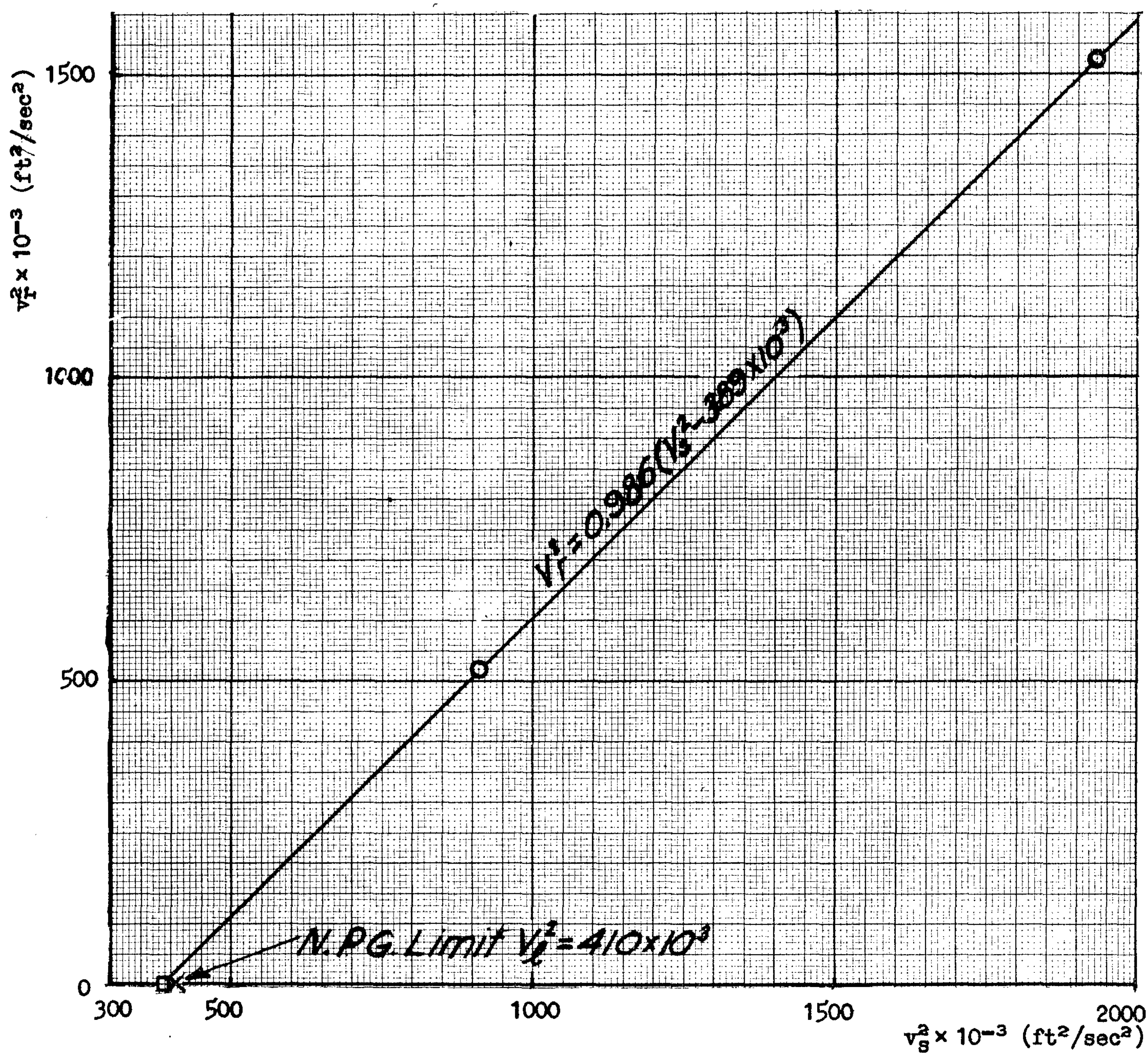

Fig. 4. Graph of data for 12-in. projectiles on 3-in. plate at normal incidence.

[redacted]

whence

$$v_l^2 = 389 \times 10^3 \text{ ft}^2/\text{sec}^2, \quad s = 0.986.$$

Therefore,

$$v_l = 624 \text{ ft/sec}, \qquad F = 36\,800,$$

$$P = 186\,000 \text{ lb/in.}^2 = 130 \text{ kg/mm}^2,$$

$$\gamma = 0.13.$$

The limit velocity v_l given in the NPG memorandum is 640 ft/sec; this corresponds to an even larger _F_-value.

These results are plotted in Fig. 4.

(_c_) _8-in. projectile on 1.95-in. plate at normal incidence._ --

Reference: NPG Memo S13-1(7)(B), Nov. 3, 1936.

Projectile: 8-in., W = 260 lb.

Plate: 1.95 in. class B (?),
ρ'(assumed) = 0.283 lb/in.^{3}, W' = 27.7 lb.

v_s	v_r	$v_s^2 \times 10^{-3}$	$v_r^2 \times 10^{-3}$	v_r(computed)
667	229	445	52.4	287
883	675	780	456	639
1205	1021	1452	1042	1030

The least square fit to these three points is represented by the equation,

$$v_r^2 = 0.970\,(v_s^2 - 360 \times 10^3),$$

whence

$$v_l^2 = (360 \pm 17) \times 10^3 \text{ ft}^2/\text{sec}^2, \quad s = 0.970 \pm 0.030.$$

It follows that

$$v_l = (600 \pm 14) \text{ ft/sec}, \qquad F = 36\,000 \pm 900,$$

$$P = (178\,000 \pm 9000) \text{ lb/in.}^2 = (125 \pm 6) \text{ kg/mm}^2,$$

$$\gamma = 0.3.$$

[redacted]

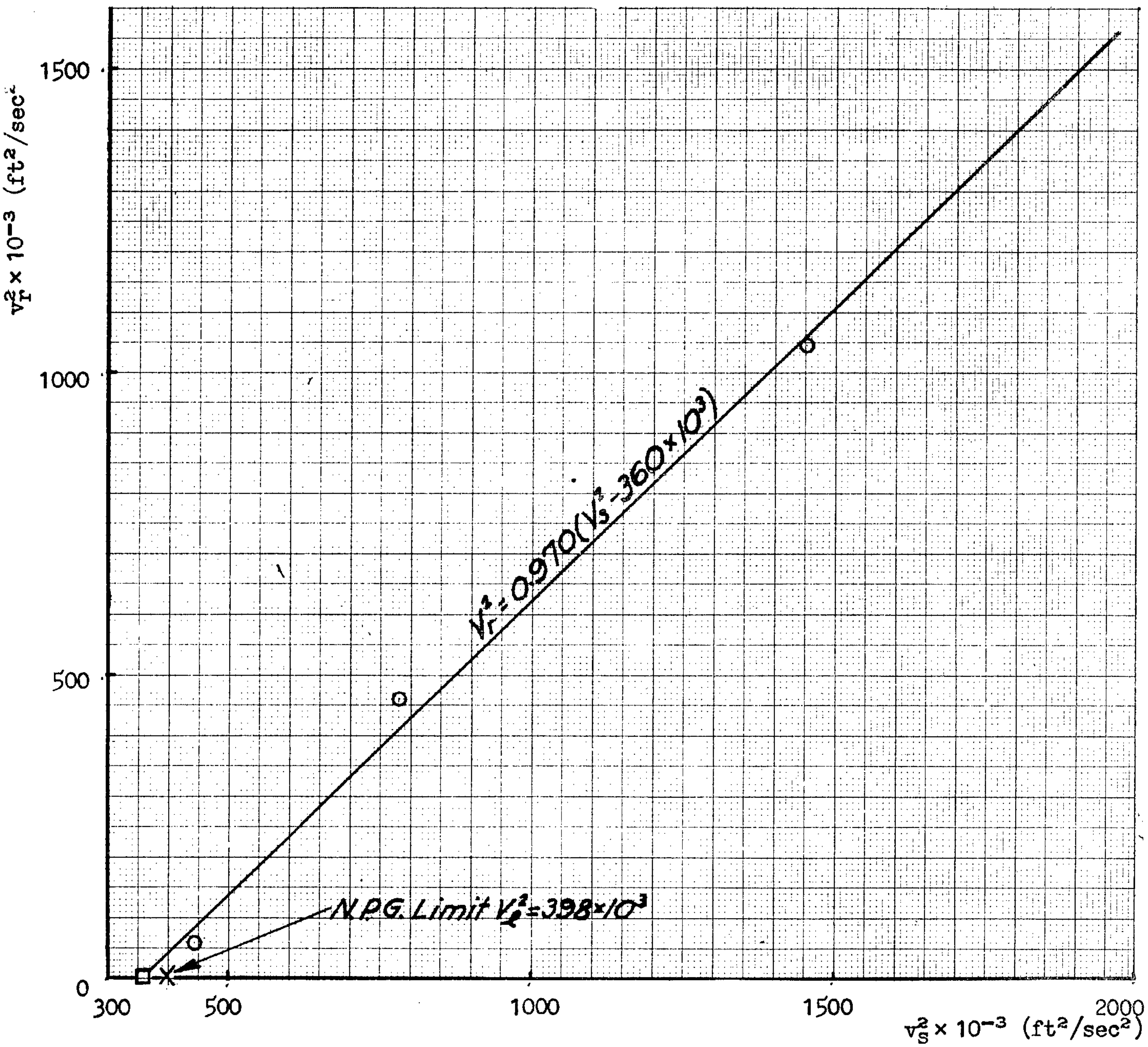

Fig. 5. Graph of data for 8-in. projectiles on 1.95-in. plate at normal incidence.

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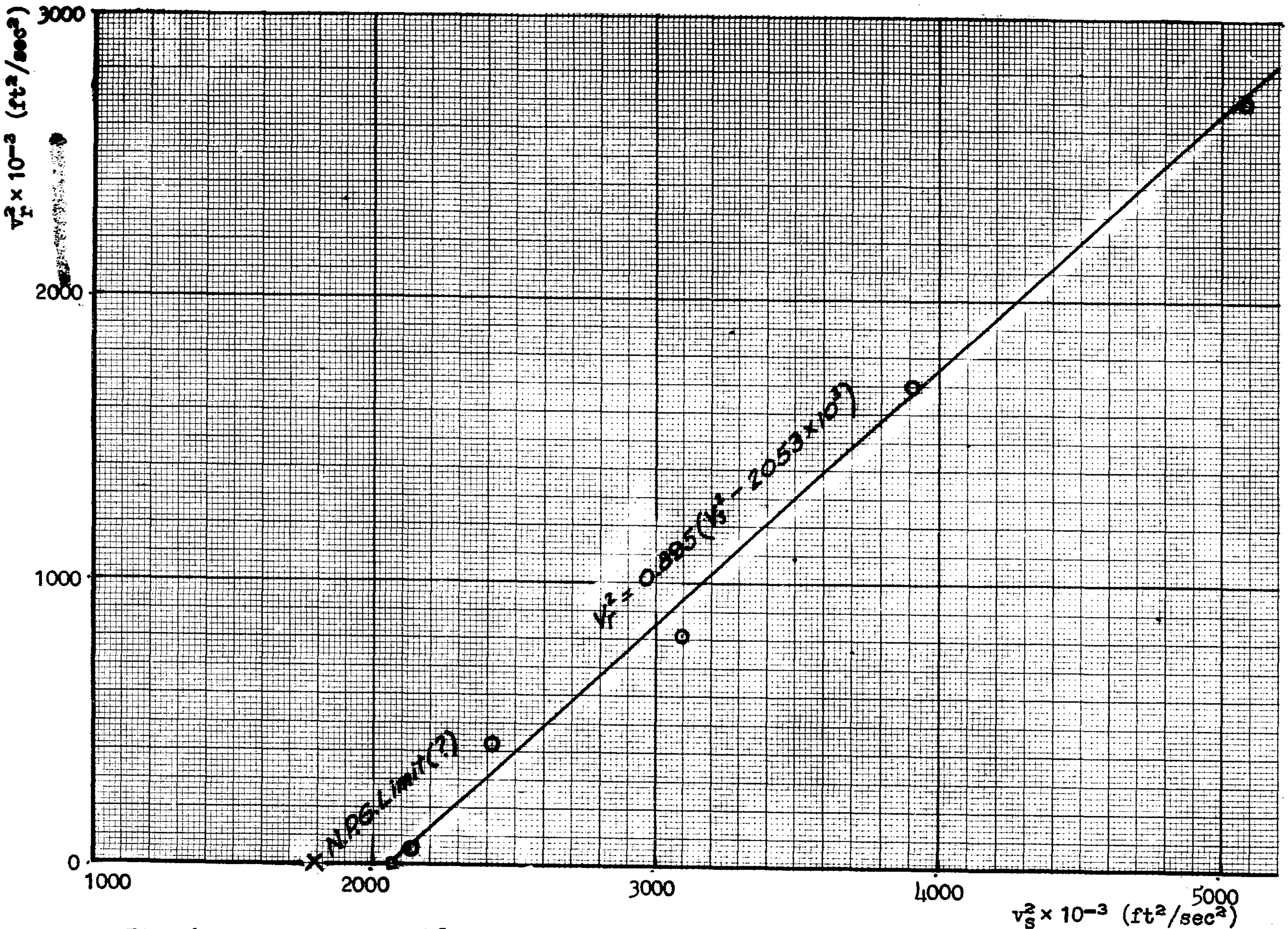

Fig. 6. Graph for 8-in./4½ cal. common projectiles on 5-in. plate at 30° obliquity.

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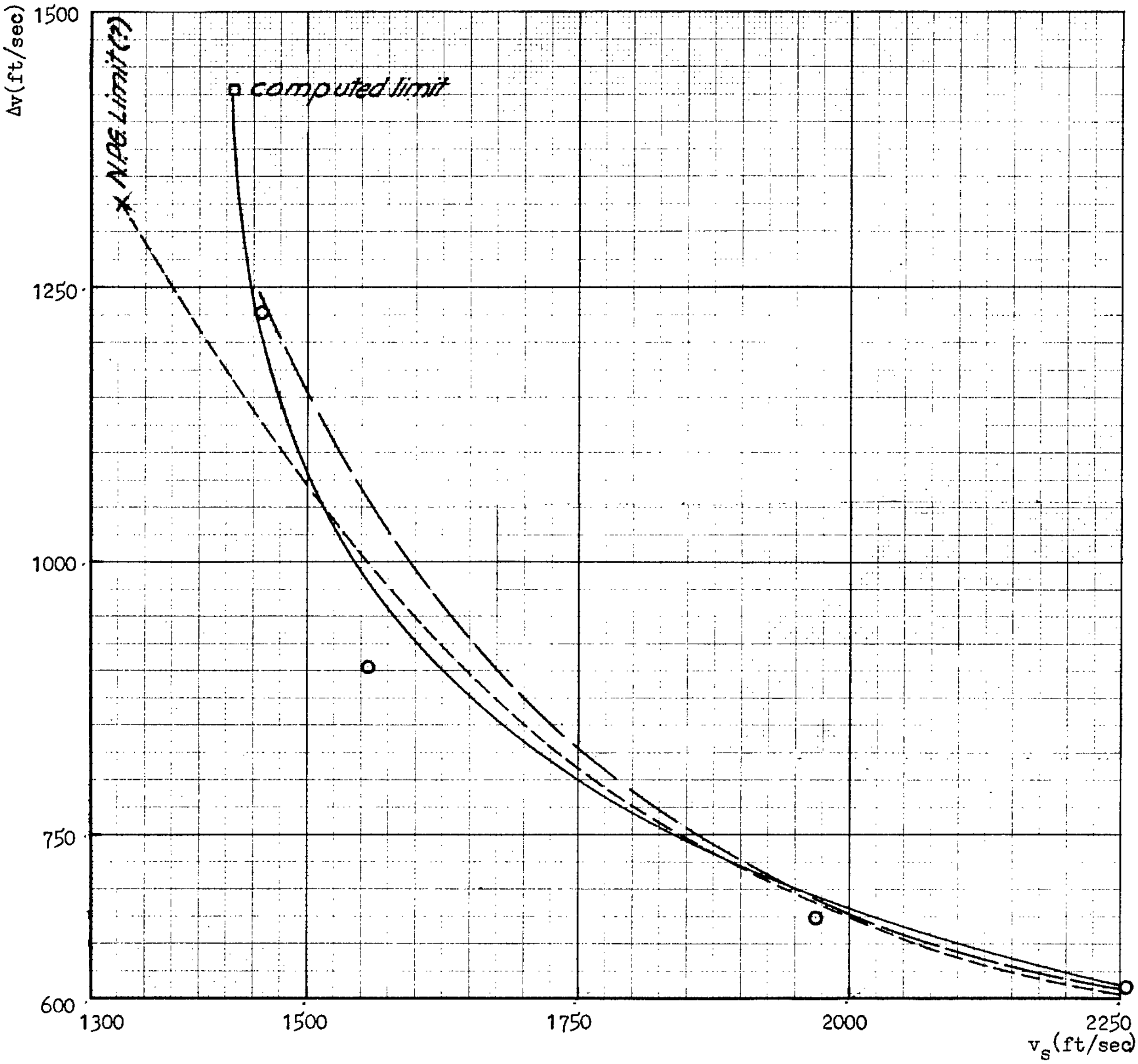
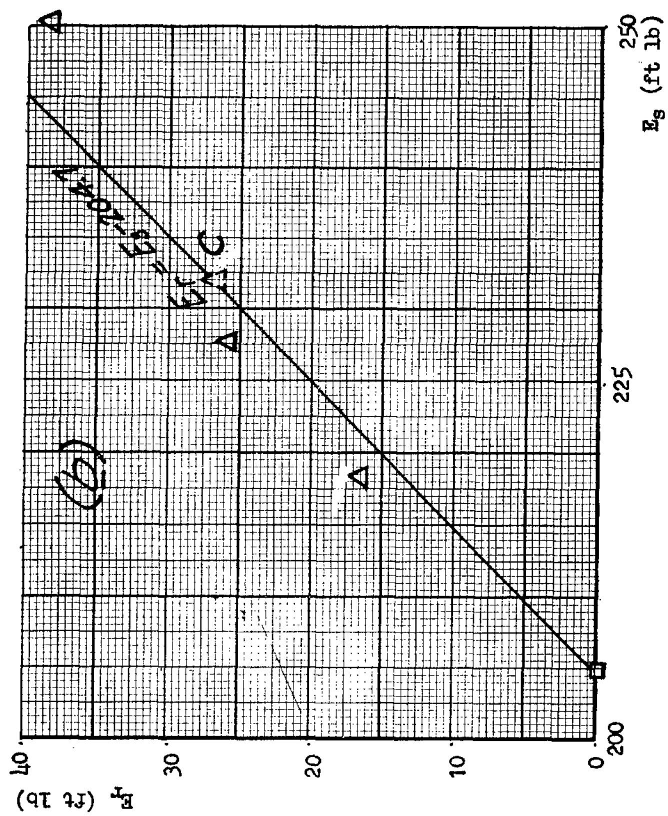


Fig. 7. Graph of data for 8-in./$4\frac{1}{2}$ cal. common projectiles on 5-in. plate at 30° obliquity; — — inked curve on photo, - - - - pencilled curve on photo,—— least square fit. These data are plotted in the manner of N.P.G. photo No. 4255.

[redacted]



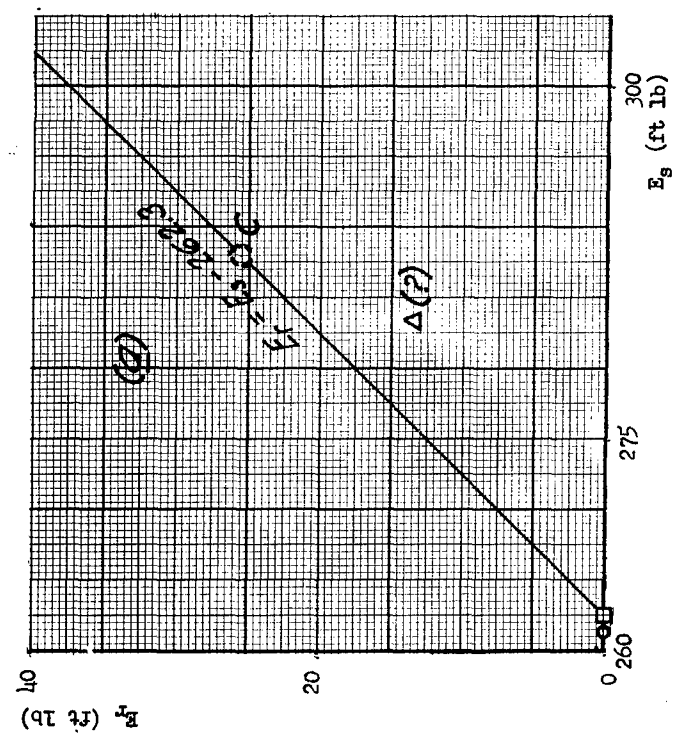


Fig. 8. Graph of data for normal incidence on $\frac{1}{4}$-in. mild steel plate; (a) caliber .2655 projectiles, (b) caliber .246 projectiles.

No confidence can be placed in this computed value of γ, because of the uncertainty in the deviation of the slope s from the critical value 1. The NPG memorandum assigns to v_l the value 631 ft/sec, corresponding to the higher F-value, 37800.

These results are plotted in Fig. 5.

(d) 8-in. projectile on 5-in. plate at 30° obliquity. --

Reference: NPG Photo No. 4255, Nov. 1930.
Projectile: 8-in./$4\frac{1}{2}$ cal. common, W = 260 lb.
Plate: 5-in.

Although the development in Sec. 1 dealt exclusively with normal incidence, it may be of interest to treat this quite complete run at 30° obliquity by the same method. For it (reading from the Photo),

v_s	v_r	$v_s^2 \times 10^{-3}$	$v_r^2 \times 10^{-3}$	v_r(computed)
1462	232	2137	54	274
1555	650	2418	422	572
1758	903	3091	815	964
1977	1304	3909	1700	1289
2253	1645	5076	2706	1645

The least square fit to these data is given by

$$v_r^2 = 0.895\,(v_s^2 - 2053 \times 10^3),$$

whence

$$v_l^2 = (2053 \pm 26) \times 10^3 \text{ ft}^2/\text{sec}^2, \qquad s = 0.895 \pm 0.018.$$

From these values it follows that

$$v_l = (1433 \pm 9) \text{ ft/sec}, \qquad F = 46500 \pm 300,$$

where the F-value is computed from Eq. (1.7) by using $v_l \cos 30^\circ = 1241$ ft/sec in place of v_l, in accordance with NPG procedure. This inferred limit velocity is considerably higher than that indicated in pencil in the copy of the Photo here used; the latter seems to be about 1330 ft/sec.

These results are plotted in Figs. 6 and 7 both in the v_r^2, v_s^2-representation here adopted and in the manner of NPG Photo No. 4255.

3. Residual velocity: small caliber

Very few data on residual velocity of small caliber projectiles are available for the purposes of this report. Those that are discussed below are by-products of experiments performed at the Naval Research Laboratory on the penetration of small plates by projectiles, the jacket weights of which had been reduced to but a small fraction of the total bullet weight. Before these data can be used in the present connection, certain reductions must be made, as outlined below.

First, the mass of the projectile is not entirely negligible in comparison with the mass of the plate; therefore, in place of the actual mass $\underline{m}$, it is necessary to employ the reduced mass,

$$\bar{m} = \frac{m}{1 + m/m_{pl}} , \qquad (3.1)$$

where m_{pl} is the mass of the target plate. The residual velocity which must then be used is the residual velocity of the projectile relative to the plate. However, since the NRL experiments give this quantity directly, the correction is here unnecessary.

Second, there is a rather considerable spread in the masses of the different projectiles used, amounting in some cases to as much as 6 percent, and the data should be reduced to that for a projectile whose mass is the average of those used in the run. But an examination of the theoretical considerations of Sec. 1 shows that the principal correction necessitated by this variation is taken care of automatically if striking and residual energies are used in the reduction; the further corrections are then of the order $\delta m'/m$ when compared to this principal correction and may therefore safely be ignored here.

(a) Caliber .2655 on $\frac{1}{4}$-in. mild steel at normal incidence. --

Reference: NRL Report No. 0-1591, Table 2.

Projectile: $d = 0.2655$ in., $W_{avg} = 103$ grains.

Plate: $\frac{1}{4}$-in. mild steel,
Brinell hardness = 110, $W_{avg} = 1819$ grains.

W	$\overline{W}$	v_s	v_r	E_s	E_r
(grains)		(ft/sec)		(ft lb)	
104	98.2	1222	475	326.9	49.2
103	97.5	1127	349	275.1	26.4
101	95.8	1108	0	261.3	0.0
				(Centroid, 287.8	25.5)

These data are so scattered that no attempt has been made at a least-square fit; instead, a 45°-line has been drawn through the centroid in the E_r, E_s-plot, the values yielded being

$$E_l = 262.3 \text{ ft lb}, \qquad v_l = 1102 \text{ ft/sec},$$

$$F = 40\,700, \qquad P = 227\,000 \text{ lb/in}^2 = 160 \text{ kg/mm}^2.$$

These data and the straight-line fit are presented graphically in Fig. 8(a).

(b) Caliber .246 on $\frac{1}{4}$-in. mild steel at normal incidence. --

Reference: Data communicated by NRL.

Projectile: $d = 0.246$ in., $W_{avg} = 87.75$ grains.

Plate: $\frac{1}{4}$-in. mild steel,
Brinell hardness = 110 ± 5, $W_{avg} = (1900 \pm 20)$ grains.

W	$\overline{W}$	v_s	v_r	E_s	E_r
(grains)		(ft/sec)		(ft lb)	
87.75	83.9	1082	301	218.2	16.9
88.5	84.6	1040	423	203.3	33.6
87.25	83.4	1108	374	227.5	25.9
87.75	83.9	1158	456	250.0	38.8
(Centroid, omitting point No. 2,				231.9	27.2)

The second point is so badly out of line that it is considered best to omit it from the reduction. The 45°-straight line through the centroid then gives

$$E_l = 204.7 \text{ ft lb}, \qquad v_l = 1049 \text{ ft/sec},$$

$$F = 38800, \qquad P = 207000 \text{ lb/in}^2 = 145 \text{ kg/mm}^2$$

This reduction is represented graphically by the curve of Fig. 8(b).

(c) Caliber .246 on 3/16-in. mild steel at normal incidence. —

Reference: Data communicated by NRL.

Projectile: d = 0.246 in.; W_{avg} = 87.8 grains.

Plate: 3/16-in. mild steel, [normal incidence;] Brinell hardness = 110 ± 5, W_{avg} = (1425 ± 15) grains.

W	$\overline{W}$	v_s	v_r	E_s	E_r
(grains)		(ft/sec)		(ft lb)	
87.75	82.7	1029	522	194.6	50.1
87.75	82.7	1008	439	186.7	35.4
88.0	82.9	967	357	172.2	23.5
				(Centroid, 184.5	36.3)

A 45°-line through the centroid yields

$$E_l = 148.2 \text{ ft lb}, \qquad v_l = 898 \text{ ft/sec},$$

$$F = 38100, \qquad P = 200000 \text{ lb/in}^2 = 140 \text{ kg/mm}^2$$

The reduction is represented in Fig. 9.

(d) Caliber .246 on $\frac{1}{4}$-in. mild steel at normal incidence. —

Reference: Data communicated by NRL.

Projectile: d = 0.246 in., W_{avg} = 85.2 grains.

Plate: $\frac{1}{4}$-in. mild steel, Brinell hardness = 150(?), W_{avg} = (1900 ± 20) grains, ρ' (assumed) = 0.283 lb/in^3, W' = 0.00336 lb.

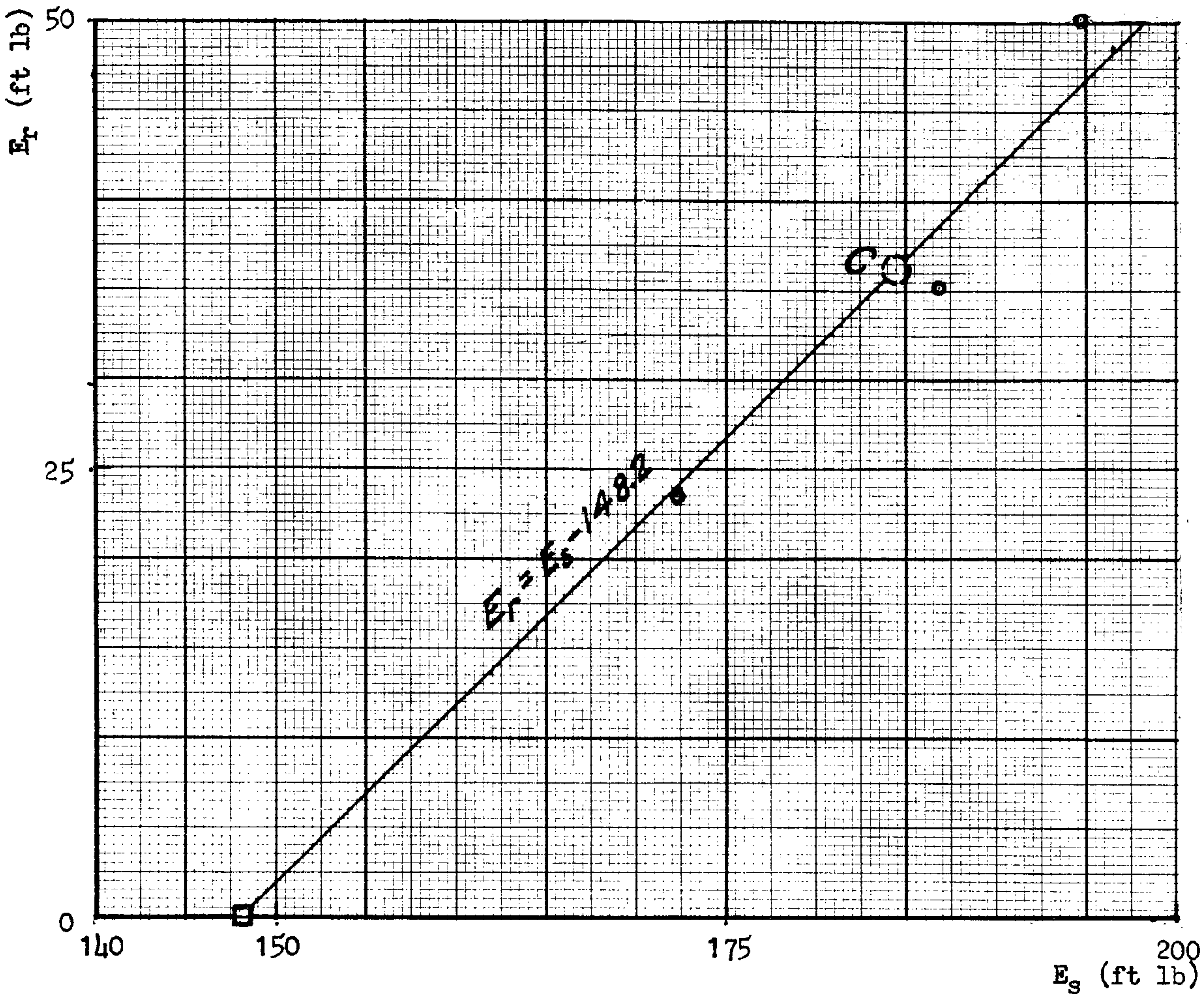


Fig. 9. Graph of data for caliber .246 projectiles on 3/16-in. mild steel at normal incidence.

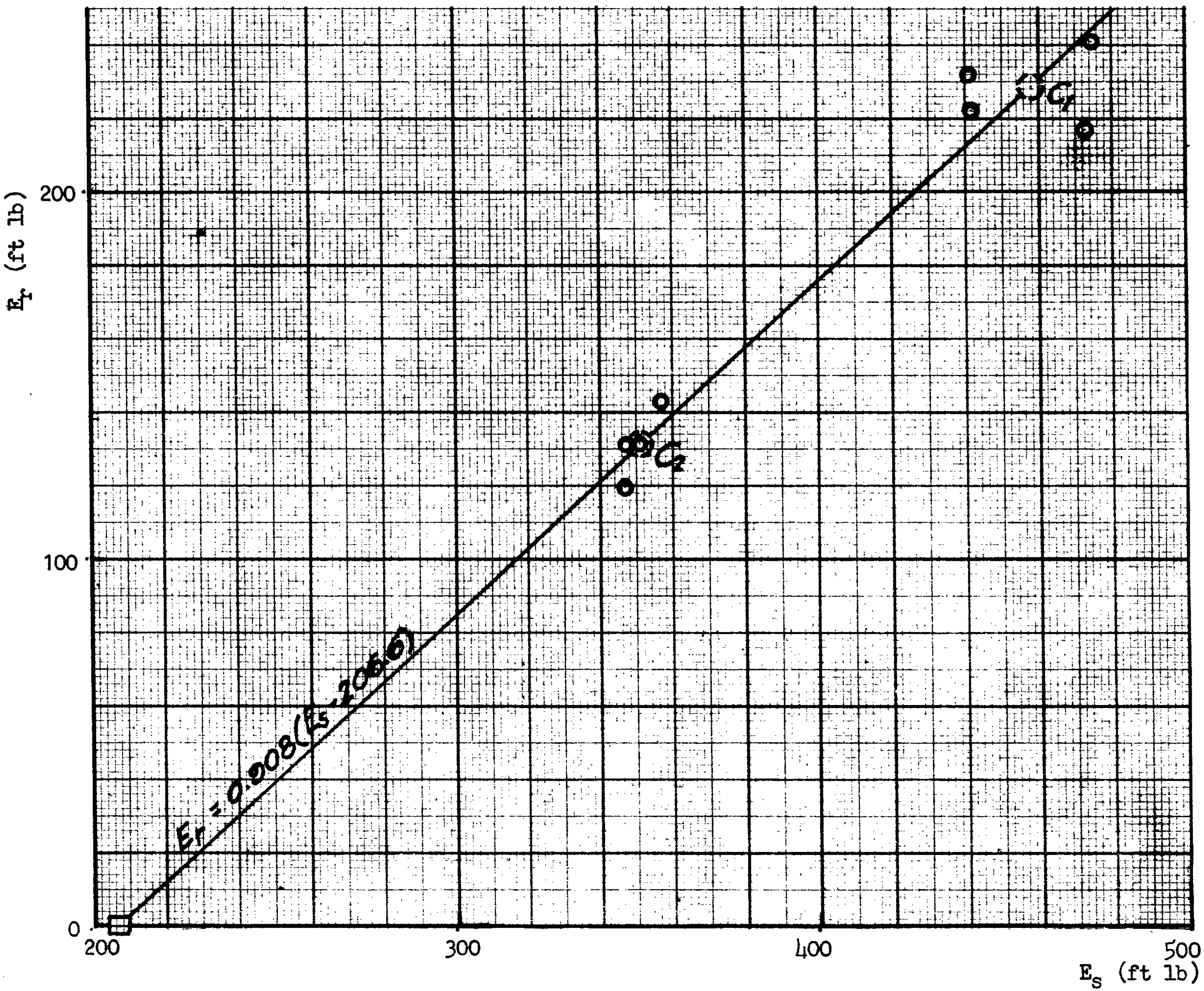


Fig. 10. Graph of data for caliber .246 projectiles on $\frac{1}{4}$-in. mild steel at normal incidence.

W	$\overline{W}$	v_s	v_r	E_s	E_r
(grains)		(ft/sec)		(ft lb)	
86.0	82.3	1612	1148	475.1	241.0
85.75	82.0	1554	1127	440.1	231.4
86.0	82.3	1554	1102	441.6	222.0
86.0	82.3	1608	1091	472.8	217.6
(Centroid of first 4 points,				457.4	228.0)
86.5	82.7	1395	880	357.5	142.3
83.75	80.2	1396	857	347.2	130.9
86.5	82.7	1373	805	346.4	119.1
(Centroid of last 3 points,				350.4	130.8)

These data points fall rather naturally into two groups, within each of which there is considerable scattering. It has therefore seemed most reasonable to pass a straight line through the two centroids of these groups, as shown in Fig. 10. This line is represented by the equation,

$$E_r = 0.908\,(E_s - 206.6),$$

whence

$$E_l = 206.6 \text{ ft lb}, \qquad v_l = 1064 \text{ ft/sec},$$

$$F = 38\,950, \quad P = 209000 \text{ lb/in}^2 = 147 \text{ kg/mm}^2,$$

$$\gamma = 0.33.$$

(e) Caliber .2655 on $\frac{1}{4}$-in. STS (homogeneous armor) at normal incidence. --

Reference:	NRL Report No. 0-1591, Table 2; supplementary data from NRL.
Projectile:	$d = 0.2655$ in.; $W_{avg} = 100$ grains.
Plate:	$\frac{1}{4}$-in. STS, Brinell hardness = 240, $W_{avg} = 1903$ grains, $\rho' = 0.282$ lb/in^2, $W' = 0.00393$ lb.

W	$\overline{W}$	v_s	v_r	E_s	E_r
(grains)		(ft/sec)		(ft lb)	
104	98.5	1543	765	521.0	128.1
101	95.9	1440	490	441.8	51.3
96.5	91.9	1316	0	353.6	0.0
98	93.3	1367	0	387.4	0.0

The straight line having the equation

$$E_r = 0.970\,(E_s - 388.9)$$

passed through the first two points. It yields

$$E_l = 388.9 \text{ ft lb}, \qquad v_l = 1359 \text{ ft/sec},$$
$$F = 49500, \quad P = 337000 \text{ lb/in}^2 = 237 \text{ kg/mm}^2,$$
$$\gamma = 0.105.$$

This reduction is represented in Fig. 11.

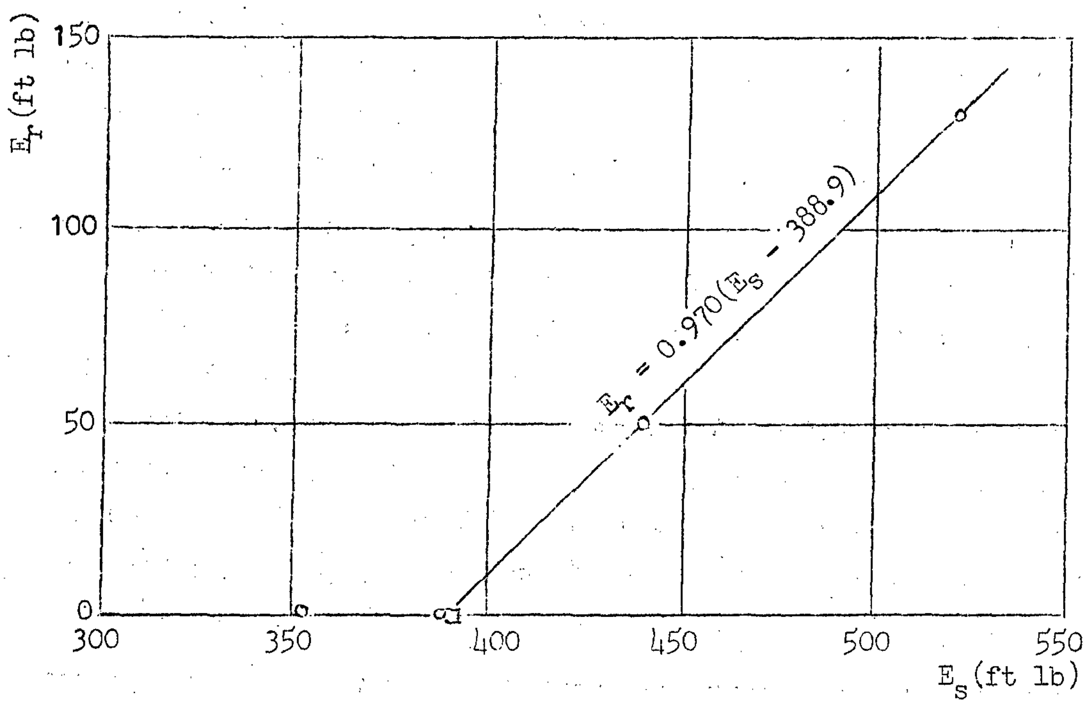


Fig. 11. Graph of data for caliber .2655 projectiles on $\frac{1}{4}$-in. STS armor at normal incidence.

4. Summary

It is shown that, on the Poncelet-Morin theory of the resistance offered a projectile in motion through a dense medium, the residual energy of the projectile on emerging from a plate is a linear function of its striking energy and (in Appendix A) that this linear dependence is characteristic of any resistance which is linear in the instantaneous energy of the projectile. It is also shown (in Appendix A) that, subject to certain limitations, the inverse relation holds; namely, if the residual energy is found experimentally to be a linear function of the striking energy, then the resistance at any stage in the penetration cycle must depend linearly on the instantaneous energy of the projectile. It is believed that the study of this dependence offers a tool which should prove of theoretical value in testing proposed mechanisms of perforation, and of practical value in enabling the determination of plate limit from even a single shot at a velocity above the limit.

Data on major caliber projectiles, furnished by the Naval Proving Ground, were analyzed in accordance with the procedure proposed in Sec. 1; it is found possible to represent these data in terms of a linear relationship between residual and striking energy, and to deduce therefrom plate limits that are in good accord with those obtained at the Naval Proving Ground from these and other data.

Data on small caliber projectiles, from the Naval Research Laboratory, are subjected to the same analysis; these data, which are by-products of a study of force-penetration relations, are found to straggle more than the major caliber data but are not inconsistent with the present hypothesis. (Small caliber tests, in which efforts are directed solely toward data of relevance here, are in progress under the auspices of Section S, Division A, NDRC and will be reported in due course by those responsible;[5/] a preliminary analysis along the present lines indicates good agreement with the theory here proposed.)

5/ Reference 2.

APPENDIX A

Extension of Poncelet Theory

Resistance a linear function of energy

It was assumed in the foregoing treatment and in the application of the Poncelet theory to armor perforation in TB-I that $\underline{a}$ and γ were parameters which depended, at most, on the physical properties of the medium and on the shape of the projectile. But many of the formal results there obtained would hold even in case these parameters depended on certain other over-all characteristics of the system -- for example, on the thickness $\underline{e}$ of the plate. Indeed, it requires but a trivial modification of the treatment to extend it to cases in which these parameters depend on $\underline{x}$, the depth of penetration. Such an extension is desirable for the sake of possible applications to inhomogeneous structures, such as composite or face-hardened armor, and because it may form a framework for a more satisfactory phenomenological description of the penetration cycle.

In order to construct such a framework broad enough to cover these applications, consider the most general equation of motion in which the resistance $\underline{R}$ is a linear function of v^2, with coefficients $\underline{\alpha}$ and $\frac{1}{2}\Gamma$ which may depend on the depth of penetration $\underline{x}$. If the air resistance is negligibly small compared with that offered by the medium, both of these functions must vanish for $x < 0$ and for $x > e + l$, where $\underline{l}$ is the total length of the projectile. In this case,

$$m\,d^2x/dt^2 = -\alpha(x) - \tfrac{1}{2}\Gamma(x)\,v^2, \tag{A-1}$$

or

$$dE/dx = -\alpha(x) - \frac{1}{m}\Gamma(x)\,E, \tag{A-2}$$

where $\underline{E}\,[=\frac{1}{2}mv^2]$ is the kinetic energy of the projectile. After penetrating to depth $\underline{x}$, the striking energy E_s of the projectile is reduced to

$$E \doteq \varepsilon^{-c(x)/m}\left[E_s - \int_0^x \varepsilon^{c(u)/m}\,\alpha(u)\,du\right], \tag{A-3}$$

where

$$c(x) = \int_0^x \Gamma(u)\,du. \tag{A-4}$$

The residual energy E_r after the projectile has completely perforated the plate is related to the striking energy E_s by

$$E_r = \varepsilon^{-c/m}(E_s - E_l), \tag{A-5}$$

where $c = c(\infty)$, and the limit energy

$$E_l = \int_0^\infty \varepsilon^{c(u)/m}\,\alpha(u)\,du. \tag{A-6}$$

Hence, if the resistance R is a linear function of the instantaneous energy E, then the residual energy E_r is a linear function of the striking energy E_s.

Inverse problem: E_r a linear function of E_s

The data on residual energy examined in this report does indicate that this energy is a linear function of striking energy, and it is therefore a matter of considerable interest to know to what extent the inverse of the foregoing theorem holds. That is, if E_r is found empirically to depend linearly on E_s, under what conditions does it follow that the resistance encountered by the projectile, at any stage of the penetration, is a linear function of its instantaneous energy E? Now it is readily shown that if the kinetic energy E at any stage depends linearly on the striking energy, then the resistance is a linear function of E -- but it can scarcely be expected that such a far-reaching conclusion could be deduced from a knowledge of the residual energy alone. The author has not been able to formulate explicitly the least restrictive conditions under which the inverse in question will hold but has contented himself with showing that the assumptions upon which the development in TB-I was based are sufficient. Precisely: If the resistance-velocity curves corresponding to any two depths of penetration are the same, to within a multiplicative factor (and independent of the thickness of the plate), and if the residual energy depends linearly on the striking energy, then the resistance at any phase of the penetration cycle is a linear function of the kinetic energy of the projectile at that phase.

In order to establish this theorem, write the equation of motion in the form[6]

$$m\,dy/dx = -\phi(x,e)\,f(y), \qquad \text{(A-7)}$$

where $y = \frac{1}{2}v^2$, and define

$$\Phi(e) = \int_0^\infty \phi(x,e)\,dx, \qquad F(y) = \int_0^y dy/f(y) \qquad \text{(A-8)}$$

The relationship between the residual specific energy y_r and the striking specific energy y_s is then defined implicitly by the integral,

$$F(y_s) - F(y_r) = \frac{1}{m}\,\Phi(e) = F(y_l). \qquad \text{(A-9)}$$

Let it now be assumed that, for all values of $y_s \geq y_l$ and for all values of e, it is found that

$$y_r = K(e,m)[y_s - y_l(e,m)]; \qquad \text{(A-10)}$$

6/ See TB-I, p. 4.

this assumption is suggested by the experimental results examined in this report (and by the preliminary analysis of the work under progress mentioned in the summary, Sec. 4). What can then be inferred concerning the dependence of $F(y)$, and thence of $f(y)$, on the specific energy $\underline{y}$?

The conditions thus imposed on $F(y)$ can be found by replacing y_r from Eq. (A-10) in Eq. (A-9) and demanding that the resulting functional equation,

$$F(y_s) - F\{K(e,m)[y_s - y_l(e,m)]\} = F[y_l(e,m)], \tag{A-11}$$

be satisfied identically in the independent variables y_s, $\underline{e}$ (and $\underline{m}$). If this identity is differentiated with respect to y_s, and the relation $dF(y)/dy = 1/f(y)$ implied by the definition (A-8) is used, it follows that

$$f(y_r) = K(e,m)\, f(y_s), \tag{A-12}$$

where y_r is given in terms of y_s, $\underline{e}$ and $\underline{m}$ by Eq. (A-10). Now differentiating Eq. (A-11) with respect to $\underline{e}$ and using Eq. (A-12) to simplify the resulting expression, we find that

$$\begin{aligned} &\frac{\partial y_l(e,m)}{\partial e} f(y_s) \\ &= f[y_l(e,m)] \left\{ \frac{\partial y_l(e,m)}{\partial e} - \frac{\partial \ln K(e,m)}{\partial e} [y_s - y_l(e,m)] \right\}. \end{aligned} \tag{A-13}$$

Inspection of this involved expression reveals that $f(y_s)$ is then a <u>linear</u> function of y_s, and hence for <u>all</u> values of $\underline{y}$ the function $f(y)$ must be of the form,

$$f(y) = a + 2by, \tag{A-14}$$

thus establishing the inverse theorem.

It was assumed in the foregoing proof that $f(y)$, and hence $\underline{a}$ and $2\underline{b}$, were independent of both $\underline{e}$ and $\underline{m}$; independence of $\underline{e}$ was essential to the proof, but independence of $\underline{m}$ played no role in it. Now it is clear that the latter assumption is eminently reasonable -- indeed much more so than the former -- and it is easy to show that it implies a restriction on the form of the coefficient $K(e,m)$ introduced in the hypothesis (A-10). On returning to the full Eq. (A-9), only part of which was used in Eq. (A-11), and replacing $\underline{F}$ in it by its value,

$$F(y) = \int_0^y \frac{dy}{f(y)} = \frac{1}{2b} \ln(1 + 2by/a), \tag{A-15}$$

we find that the residual and limit specific energies y_r and y_l are given by

$$y_r = \varepsilon^{-\frac{2b}{m}\Phi(e)}(y_s - y_l), \qquad \text{(A-16)}$$

$$y_l = \frac{a}{2b}\left(\varepsilon^{\frac{2b}{m}\Phi(e)} - 1\right). \qquad \text{(A-17)}$$

Comparison of the first of these equations with Eq. (A-10) shows that, if this work is to be valid, K is restricted to the form,

$$K(e,m) = \varepsilon^{-\frac{2b}{m}\Phi(e)}. \qquad \text{(A-18)}$$

This dependence of the slope on the mass of the projectile gives, in principle, a check on the validity of the hypotheses used in the foregoing proof of the inverse theorem -- but one that is practicable only in the case of a careful series of tests using otherwise equal bullets with a fairly wide range of variation in mass, such as might be achieved by the inclusion of a few Carboloy slugs.

The case here considered leads back to the Poncelet formulas (1.4) and (1.5) on taking $\phi(x,e)$ as the area of impression $A(x,e)$ and setting b equal to $\frac{1}{2}\gamma\rho'$.

Extension of Poncelet-Morin theory

Returning to the general development given at the beginning of this appendix, we obtain an extension of the Poncelet-Morin basic theory on setting

$$\mathcal{a} = A(x,e)\,a(x,e), \qquad \Gamma = A(x,e)\,\rho'\gamma(x,e), \qquad \text{(A-19)}$$

where the shatter strength a and the inertial coefficient γ may vary with depth x of penetration, as well as with the thickness e of the plate. The auxiliary quantity $c = c(\infty)$ -- Eq. (A-4) -- is then

$$c = \int_0^\infty \Gamma(x)\,dx = \int_0^\infty \gamma(x,e)\,d\rho' V(x,e) = \bar{\gamma}(e)m', \qquad \text{(A-20)}$$

where $\bar{\gamma}(e)$ is the volume-of-impression average of $\gamma(e,x)$. The formulas (A-5) and (A-6) are then

$$E_r = \varepsilon^{-\bar{\gamma}(e)m'/m}(E_s - E_l), \qquad \text{(A-21)}$$

$$E_l = \int_0^\infty \varepsilon^{c(u)/m}\,a(u,e)\,dV(u,e). \qquad \text{(A-22)}$$

fra

C [redacted]

Work of L. Gabeaud

In papers published in 1935 in the Memorial de l'artillerie française, L. Gabeaud carried out theoretical investigations on armor perforation that amount in essence to special cases of the general theory here considered, and applied them to the problem of predicting limit velocity; but, since we are here more concerned with the problem of residual, rather than limit, velocity, discussion of this work will be deferred to a later report. Of some interest in the present connection, however, is the fact that Gabeaud attempts to take into account the contribution of friction to the resistance encountered by the projectile; it will suffice for present purposes to state that, if it be assumed that the coefficient of friction is independent of velocity over the range in question, the resulting theory can be subsumed under the theory here developed; hence also in this case, the residual energy will be a linear function of the striking energy.

APPENDIX B

Addendum by A. H. Taub and C. W. Curtis

Numerous experimental and theoretical contributions to the understanding of the mechanism of armor penetration have been made during the two years which have elapsed since H. P. Robertson prepared the report to which this is an addendum. These permit an assessment of the validity of the restricted [Eq. (1.1)] and generalized [Eq. (A-1)] Poncelet force equations. It now appears that although the former is not satisfactory, it makes predictions which are qualitatively in agreement with many experimental results, and it is possible that complete quantitative correlation can be obtained with the latter.

Consider the expression for the limit energy based on the assumption of the restricted Poncelet force and normal impact [Eqs. (1.9) and (1.12)], namely:

$$E_l = \frac{\pi a e d^2}{4}\,\frac{(\varepsilon^{z} - 1)}{z} = \frac{\pi a e d^2}{4} f(z). \qquad \text{(B-1)}$$

The function $f(z)\left[z = \gamma \frac{m'}{m} = \gamma\rho \frac{\pi d^3}{4m}\frac{e}{d}\right]$ is a slowly varying function of e/d and to a first approximation may be considered a constant. This is so since the values of γ, as obtained from the slopes of the residual energy-striking energy curves, are quite small. Physically this means that the forces due to the motion of the plate material are negligibly small in comparison with the static forces necessary for the production of the hole. If the inertial forces are completely neglected, $f(z) = 1$.

Since f(z) is a slowly varying function of e/d, the predictions of the restricted form of the Poncelet theory are essentially those of a theory of penetration in which it is assumed that the resisting force is due to a constant pressure a in the plate. In particular, if f(z) is taken as unity, E_l is the energy necessary for overcoming the pressure a. This pressure has been interpreted by Bethe7/ as the hydrostatic pressure necessary to expand a cylindrical hole in the plate uniformly by displacing the plate material laterally until the radius of the hole is equal to that of the projectile.

Under certain conditions the mechanism of penetration is obviously not one of overcoming a constant hydrostatic pressure, and it is not to be expected that the dependence of E_l on e and d will be that just given. For example, when a plug is formed, shearing stresses are involved. These act over the lateral surface πed of the cylinder punched out of the plate by the projectile and one would expect E_l to be proportional

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7/ H. A. Bethe, "Attempt of a theory of armor penetration," Frankford Arsenal Rept. (1941), pp. 13 and 16.

to e^2d rather than to ed^2. This form has been verified for a flat-nosed projectile striking a plate whose thickness is one caliber or less.8/

Another clear example of the failure of the constant hydrostatic pressure mechanism occurs for extremely thin plates (thickness less than 0.25 caliber) even when plugs are not produced. Here again E_l is proportional to 9,10/ e^2d. It has been suggested for this case, where petals are formed, that the main part of the energy of the projectile is expended in the bending back of the petals. For a thin plate the width of the petals is the same as the thickness of the plate, while in the case of a thick plate the petal width is only a small fraction of the plate thickness. The manner in which the energy due to petal formation may be taken into account in the latter case will be described later.

One might expect that the constant pressure idealization would be most likely to apply to the case of a sharp-nosed projectile striking a thick and relatively soft homogeneous plate. Even under such conditions, however, experimental values for E_l show that this assumption is not entirely justified. This is most readily seen from a graph of the "average pressure" $\underline{P}$ as a function of plate thickness.

This average pressure is defined in terms of the limit energy by the equation,

$$P = E_l / \tfrac{1}{4}\pi d^2 e, \qquad \text{(B-2)}$$

and is equal to $\underline{a}$ for the constant pressure assumption. The Poncelet force equation leads to

$$P = a f(z). \qquad \text{(B-3)}$$

In Fig. 12 there is given a series of experimental values for $\underline{P}$ which covers a wide range in plate thicknesses. The projectile used was an unjacketed caliber .30 AP M2 core whose nose had an approximate radius of ogive of 3.6 calibers. This was fired against homogeneous plate of Brinell hardness number 255 ± 7.

8/ C. Zener and J. E. Holloman, "Mechanism of armor penetration, first partial report," Watertown Arsenal Rept. No. 710/454, p.22.

9/ "The penetration of homogeneous armor by uncapped projectiles at 0° obliquity," U.S. Naval Proving Ground Rept. No. 1-43, p.16.

10/ The Ballistic Research Group, Princeton University, The ballistic properties of mild steel, including preliminary tests on armor steel and dural, NDRC Report A-111 (OSRD No. 1027), p. 49.

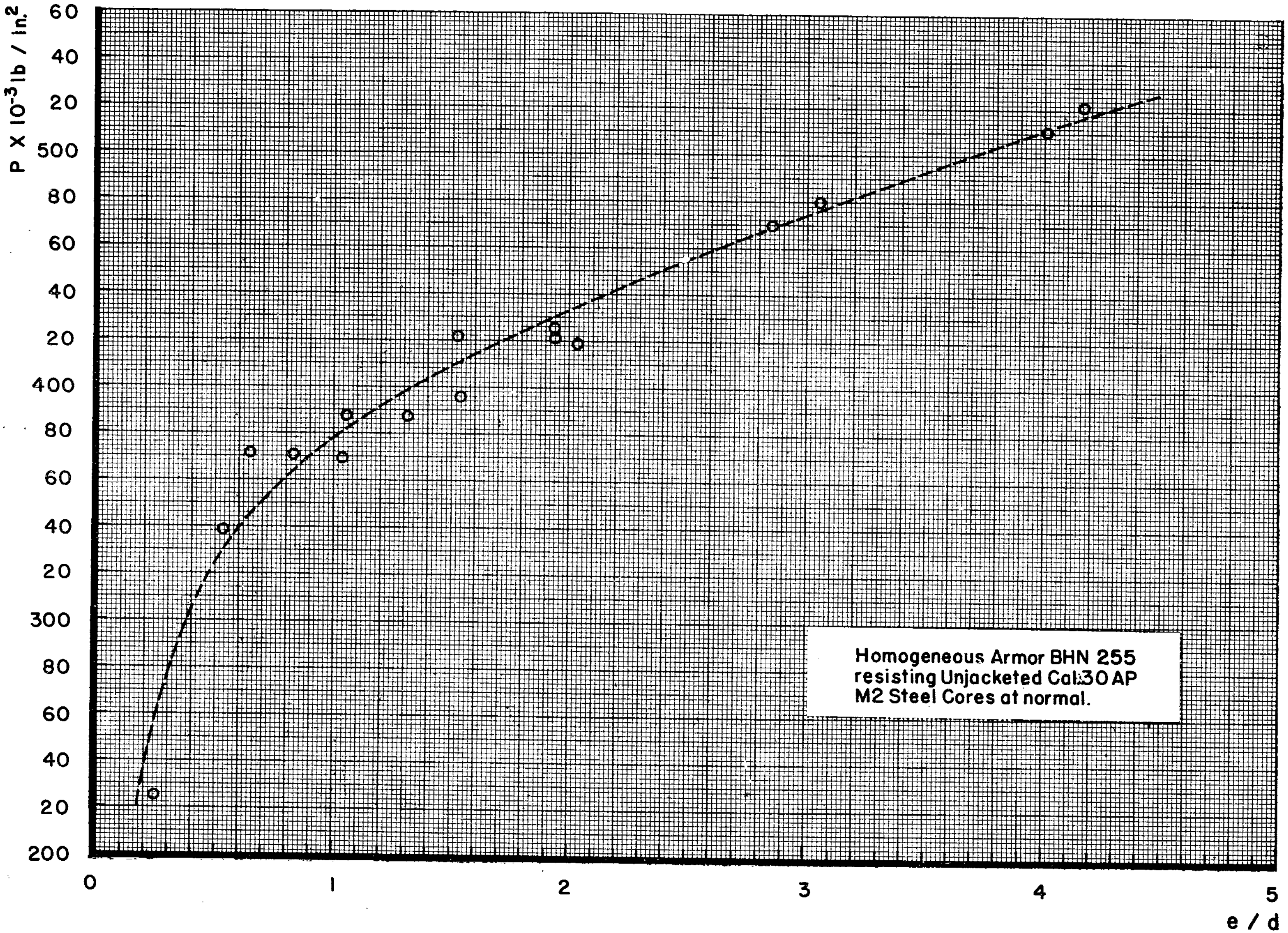

Fig. 12. AVERAGE PRESSURE OF PLATE — PROJECTILE REACTION AS A FUNCTION OF PLATE THICKNESS.

~~Confidential~~

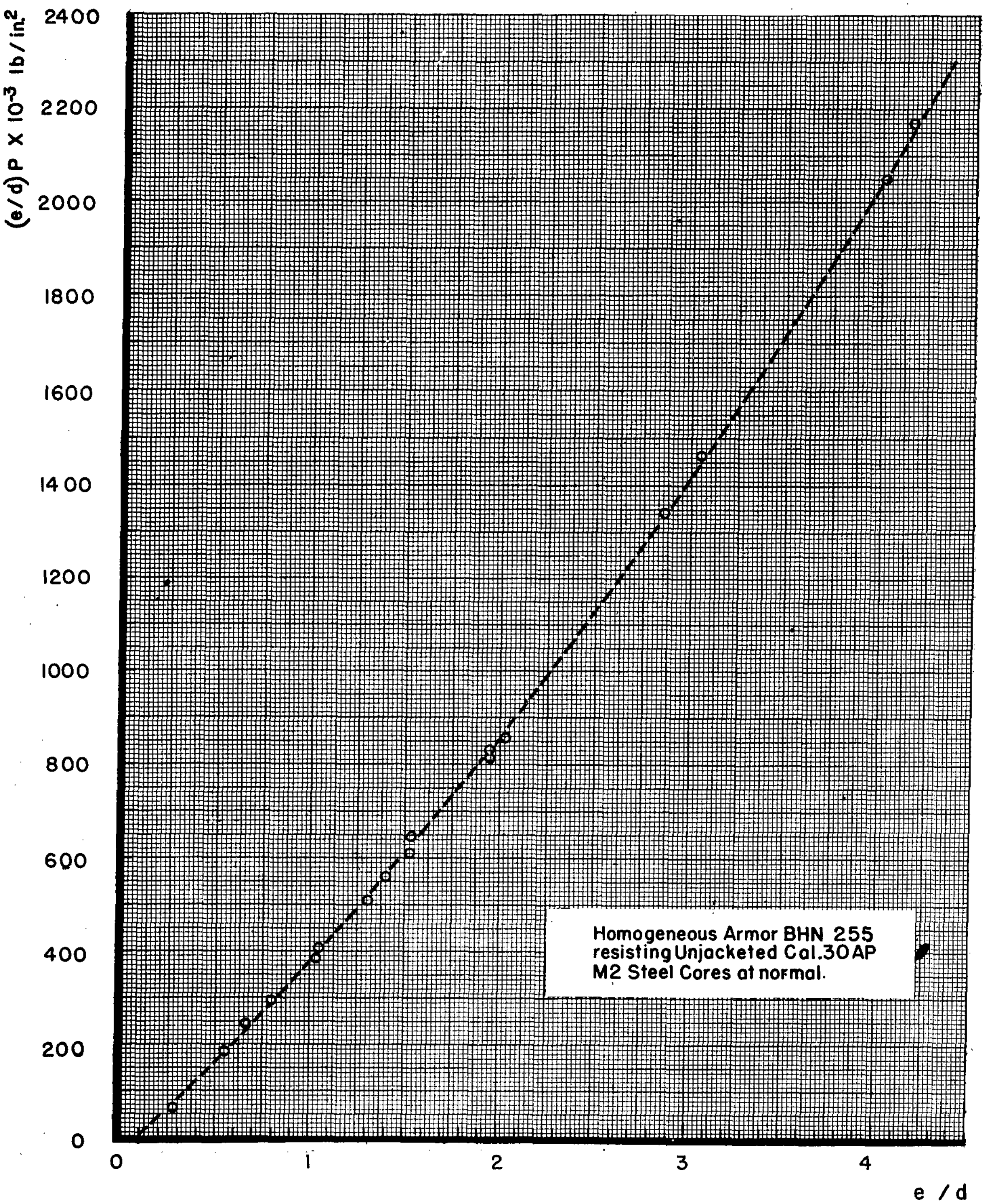

Fig. 13. SPECIFIC LIMIT ENERGY AS A FUNCTION OF PLATE THICKNESS.

~~Confidential~~

Clearly P does not remain constant;[11] furthermore, any curve representing these data must be concave downward while the form of $f(z)$ is such as to produce an upward curvature. Hence not only is the constant pressure theory untenable, but the correction introduced by the restricted form of the Poncelet theory is in the wrong direction.

A much better representation of the data is given by the following modification of Eq. (B-1).

$$E_l = \frac{\pi a}{4}[ed^2 - bd^3]f(z) \tag{B-4}$$

or

$$\frac{e}{d}P = a\left[\frac{e}{d} - b\right]f(z), \tag{B-5}$$

where b is a constant of the order of magnitude of 1/10. The data of Fig. 12 are replotted in Fig. 13 where $(e/d)P$ is taken as the ordinate and e/d as the abscissa. The smooth curve represents Eq. (B-5) in which the parameters a, b and γ have been adjusted to fit the experimental data.

This modified form of the limit energy equation and its possible physical explanation were proposed by members of the staff of the Naval Proving Ground.[12] It was assumed that the Bethe theory is valid while the projectile is in the main body of the plate, but that the mechanism of failure changes to a petalling type in a narrow region of thickness t at the back face. In this petalling region the energy absorbed is taken as proportional to t^2d, a form that is valid for this type of failure in the case of extremely thin plates. With the additional assumption that for thick plates t is independent of plate thickness and is directly proportional to the caliber of the projectile, the limit energy equations [(B-4) and (B-5)] result.

If merely static forces are considered, the energy absorbed during penetration to a depth $e-t$ is given by $\frac{1}{4}\pi ad^2(e-t)$, while the proposed expression for the absorption in the petalling region is $\pi a(\pi t^2 d/6)$. The limit energy is the sum of these two expressions; so, with $t = kd$, where k is a constant,

$$E_l = \frac{\pi a}{4}\left[ed^2 - k\left(1 - \frac{\pi k}{6}\right)d^3\right],$$

11/ Values of P obtained for the cores when they are fired as jacketed ammunition are much more nearly constant than when the cores are used bare. See also C. Zener and E. Peterson, "Mechanism of armor penetration, second partial report," Watertown Arsenal Rept. No. 710/492, p. 16.

12/ Reference 9.

which is identical with Eq. (B-4) when $f(z)$ is assumed to be unity and $\underline{b}$ is set equal to $k(1-\pi k/6)$.

Thus, by taking into account the change in the mechanism of plate reaction near the back face of the plate, one is led to expressions for the limit energy and average pressure in qualitative agreement with experiment. The agreement is only qualitative, however, for, despite the fact that Eq. (B-2) adequately represents the data within the accuracy of the measurements, the value of $\underline{\gamma}$ required is approximately four times the value obtained independently from the slopes of the residual energy-striking energy curves. This suggests that taking into account only the back edge effect due to petalling is inadequate, and that perhaps another approach is required in which both "front edge and back edge effects" are included.

The generalized Poncelet equation given in Appendix A of this report allows such an approach, since edge effects can be taken into account by appropriately choosing $\alpha(x)$ and $\Gamma(x)$. On the other hand, in the restricted form of the Poncelet equation, edge effects of the type mentioned are not included. This failure of the restricted form to consider edge effects correctly may also be seen as follows. It predicts that the limit energy for a composite plate, made up of two similar plates of thicknesses e_1 and e_2, is

$$E_{l_c} = E_{l_1} + \varepsilon^{z_1} E_{l_2},$$

where E_{l_1} and E_{l_2} are limit energies of the first and second plates, respectively; and

$$z_1 = \frac{\gamma\rho\pi d^3}{4m}\frac{e_1}{d}.$$

This can be shown to be equal to the limit energy of a single plate of thickness $e_1 + e_2$. However, it is well known that the limit energy of a composite plate is less than that of a single plate of the same thickness. Clearly the difference between the limit energies must arise because of edge effects at the intermediate faces of the composite plate.

At the present time there is no complete physical theory of armor penetration that is quantitatively consistent with all the known facts; but for limited changes in the variables involved and under restricted conditions, limit energy formulas are available that are adequate for practical purposes. A phenomenological representation of the projectile-plate reaction by means of a generalized Poncelet force equation now seems reasonable. This remains the only practical type of equation proposed that involves a consideration of the inertial forces. Owing to the inclusion of inertial effects, even the restricted one qualitatively predicts correctly all of the following observed results:

(i) The residual energy-striking energy curve is a straight line whose slope decreases with increase in plate thickness. [See Eq. (4.4).]

(ii) A small increase in the limit energy results from a decrease in the mass of the projectile because of the dependence of f(z) on m.

(iii) The limit energy increases with a decrease in the radius of the ogive of the projectile because of the dependence of γ on nose shape.

(iv) An upward curvature exists for the (e/d)P-versus-e/d line. This results from the dependence of f(z) on e.

(v) Projectiles fired against homogeneous plate shatter at high but not at low velocities, and a decrease in the shatter velocity results from a decrease in the radius of the ogive of the projectile. Such behavior would result from a force of the Poncelet type because of its velocity dependence.